B.Sc Part I (Honors)

Relation Between The Roots and Co-efficients.

Q-1: Establishes relations between the roots and Coefficients.

Ans:— Let us consider the general equation of n^{th} degree as

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \text{——} + a_{n-1}x + a_n = 0 \quad \text{—— (1)}$$

and its roots be $\alpha_1, \alpha_2, \alpha_3, \text{——} \alpha_n$

Then $f(x) = k(x-\alpha_1)(x-\alpha_2)(x-\alpha_3) \text{——} (x-\alpha_n)$ —— (2)

Comparing the Coefficients of x^n from (1) and (2)

We get $a_0 = k$

So by (2) $f(x) = a_0(x-\alpha_1)(x-\alpha_2)(x-\alpha_3) \text{——} (x-\alpha_n)$

$$= a_0\{x^n - (\alpha_1+\alpha_2+ \text{——} +\alpha_n)x^{n-1} + (\alpha_1\alpha_2+\alpha_1\alpha_3+ \text{——} + \alpha_{n-1}\alpha_n).x^{n-2} - (\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_3\alpha_4+ \text{——} +\alpha_{n-2}\alpha_{n-1}\alpha_n) \times x^{n-3} \text{——} + (-1)^n\alpha_1\alpha_2\alpha_3 \text{——} \alpha_n\}.$$

Comparing the coefficients of like powers of x both sides, we get

$$\sum\alpha_1 = \alpha_1+\alpha_2+ \text{——} +\alpha_n = (-1)^1\frac{a_1}{a_0}$$

$$\sum\alpha_1\alpha_2 = \alpha_1\alpha_2+\alpha_1\alpha_3+ \text{——} +\alpha_{n-1}\alpha_n = (-1)^2\frac{\alpha_2}{a_0}$$

$$\sum\alpha_1\alpha_2\alpha_3 = \alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_4+ \text{——} +\alpha_{n-2}\alpha_{n-1}\alpha_n = (-1)^3\frac{a_3}{a_0}$$

$$\alpha_1\alpha_2\alpha_3 \text{——} \alpha_n = (-1)^n\frac{a_n}{a_0}.$$

Example:-(1) The equation $x^4+4x^3-2x^2-12x+9=0$ has two pairs of equal roots. Find them.

Ans:— Let $\alpha, \alpha, \beta, \beta$ be roots of given equation.

Then $\alpha+\alpha+\beta+\beta = -4$

$$\Rightarrow 2\alpha+2\beta = -4 \Rightarrow \alpha+\beta = -2 \quad \text{——— (1)}$$

$$\alpha.\alpha+\alpha.\beta+\alpha.\beta+\alpha.\beta+\alpha.\beta+\beta.\beta = -2$$

$$\therefore \alpha^2+4\alpha\beta+\beta^2 = -2 \quad \text{——— (2)}$$

$\alpha\beta^2+\alpha\beta^2+\alpha^2\beta+\alpha^2\beta=12$

$\Rightarrow 2\alpha\beta^2+2\alpha^2\beta=12 \Rightarrow \alpha\beta(\alpha+\beta)=6 \longrightarrow (3)$

and $\alpha\alpha\beta\beta=9 \Rightarrow \alpha^2\beta^2=9 \Rightarrow \alpha\beta=\pm 3 \longrightarrow (4)$

By (2) $(\alpha+\beta)^2+2\alpha\beta=-2$

$\Rightarrow 4+2\alpha\beta=-2$ (By (1))

$\Rightarrow 2\alpha\beta=-6 \Rightarrow \alpha\beta=-3 \longrightarrow (5)$

By (1) & (5), $\alpha+\beta=-2$ and $\alpha\beta=-3 \longrightarrow (6)$

These values satisfy (3)

We have $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=4+12=16$

$\therefore \alpha-\beta=\pm 4$ and by (6) $\alpha+\beta=-2$

Solving ~~these~~ these two equations, we get

$$\left.\begin{matrix}\alpha=1\\ \beta=-3\end{matrix}\right\} \text{ and } \left.\begin{matrix}\alpha=-3\\ \beta=1\end{matrix}\right\}$$

$\therefore$ 1, 1, −3, −3 are four roots of given equation.

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